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Reverse Balking and Reneging in Finite Buffer Markovian Queuing System

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Abstract—*Most of the models studied in queuing theory don't assume impatient customers. However it is well-known that customers are hard pressed for time. There is also a new concept of reverse balking where customers go by the size of the crowd. In this paper, we analyze a finite buffer queuing system with impatience which is dependent on the state of the customers and reverse balking. We present steady state probabilities and also a number of performance measures.*

Keywords*: Finite capacity, Queuing Model, Reneging, Reverse Balking, Steady-State Solution.*

1. INTRODUCTION

Waiting lines or queues are familiar phenomena, which we observe quite frequently in our daily life. The queuing theory has been applied to a variety of business situations. The customer's impatience in queuing systems is an important field of research in Queuing Theory. In queuing system by balking, we mean the phenomena of customers arriving for service into a non-empty queue and leaving without joining the queue. Even if a customer joins the queue, it is logical to expect that the customer has a patience time beyond which he is not willing to wait. If service is not over within this patience time, the customer will leave the system also for good. The phenomenon of a customer joining the queuing system and leaving it before completing service is known as reneging [4].

In the queuing models reneging and balking are the functions of either system size or queue length. Larger is the system size more is balking and similar is the case of reneging. But when it comes to the sensitive businesses like investment, customers invest with the firms having large number of customers with them. Thus, the probability of joining of customers in such firms is high. Therefore in the businesses like customer investment, the probability of balking will be low when the system size is more and vice-versa, which is the balking in reverse sense and this, is called as *Reverse Balking*.

In this paper, we have introduced the concept of reverse balking in a finite buffer Markovian queuing model with considering the reneging phenomenon which is very relevant to real situations in queuing theory.

The subsequent sections of this paper are arranged as follows: section 2 contains a brief review of the literature. Section 3 and 4 contain model description and derivation of steady state probabilities. In section 5, we discuss a numerical example. We conclude in section 6

2. REVIEW OF LITERATURE:

One of the earliest works on reneging was by Barrer [6] where he considered deterministic reneging with single server Markovian arrival and service rates. Haight [7] studies a single-server Markovian queuing system with reneging. Haghighi et al. [4] analyzed a multi-server queuing model with balking and reneging. Steady state distribution of the number of customers in the system was obtained. Choudhury & Medhi [3] considered an M/M/k model with the restriction that customers may balk from a non-empty queue as well as may renege after they join the queue. Closed form expressions of a number of performance measures were derived. Choudhury & Medhi[4] analyzed a multi-server Markovian queuing system under the assumption that customers may balk as well as renege. Explicit closed form expressions were presented. A numerical example with design aspects was also discussed to demonstrate results derived. Choudhury and Medhi[5] also analyzed a single server finite buffer Markovian queuing model M/M/1/k with the additional restriction that customers may balk as well as renege. Choudhury & Medhi, [6] considered a finite buffer multi server queuing system with balking along with position dependent reneging. Explicit closed form expressions of a number of performance measures were presented.

Jain et al [14] developed and introduced the concept of reverse balking in a single server Markovian queuing system having finite capacity. The steady-state solution of the model was obtained and different measures of effectiveness were derived. Sensitivity analysis of the model was also performed. Kumar et al. [15] considered a single server finite capacity feedback queuing system with reverse balking. . Som& Kumar [17] also considered a finite capacity Markovian queuing system with two heterogeneous servers, reverse balking and reneging. The stationary system size probabilities were obtained.

3. ASSUMPTIONS OF THE MODEL:

In this paper, we shall deal with the M/M/1/k model with balking. The assumptions of the model as follows:

a) Arrivals are described by Poisson probability distribution and the inter-arrival times are exponentially distributed with parameter λ.

b) There is only one server and the service times are exponentially distributed with parameter μ.

c) The system capacity is restricted to k i.e. the capacity of the system is finite.

d) The queue discipline is 'First-Come, First-served'.

e) Here we assume that the balking probability of an arriving customer is $\left\{1-\frac{1}{(a-nb)}\right\}$and probability of joining a customer is $\frac{1}{(a-nb)}$ where the constant 'a' and 'b' have to choose in such a manner that $(a-nb)>0$ i.e. $n<\frac{a}{b}$, (n = 0,1,2,…..,k).

f) Customers joining the system are assumed to be of Markovian reneging type. We shall assume that on joining the system, the customer is aware of its state in the system. Consequently, the reneging rate is modeled as a function of the customer's state in the system. In particular, a customer who is at state *n* will be assumed to have random patience time following exp (ν_n). Here, $\nu_n$is the reneging rate when state of the system is *n*. We considered ν_n as

$$\nu_n = \nu^n \quad ; n = 1,2,\ldots\ldots,k$$

Here we also assume that $\nu > 1$. Our aim behind this formulation is to ensure that higher the current state of a customer, higher is the reneging rate. Hence we considered the reneging is position dependent.

4. THE STEADY STATE PROBABILITIES:

In this section, the steady state probabilities are derived by the Markov process method. Let $p_n$denotes the probability that there are 'n' customers in the system. The steady state equations are

$$(\mu+\nu)p_1 = \frac{\lambda}{a}p_0 \quad \text{.....(1)}$$

$$\left\{\mu+\frac{\nu(\nu^{n+1}-1)}{\nu-1}\right\}p_{n+1}+\left[\frac{\lambda}{a-(n-1)b}\right]p_{n-1} = \left(\mu+\frac{\nu(\nu^n-1)}{\nu-1}+\frac{\lambda}{a-nb}\right)p_n$$

$$; \quad n = 1,2,\ldots, k\text{-}1 \quad \text{….(2)}$$

$$\left[\frac{\lambda}{a-(k-1)b}\right]p_{k-1} = \left\{\mu+\frac{\nu(\nu^k-1)}{\nu-1}\right\}p_k ; n = k \quad \text{... (3)}$$

Solving recursively, we get

$$p_n = \left[\frac{\lambda^n}{\prod_{r=1}^{n}\left(\mu+\frac{\nu(\nu^r-1)}{\nu-1}\right)\prod_{r=0}^{n-1}(a-rb)}\right]p_0 \, ; n = 1,2,\ldots,k \quad \text{….(4)}$$

Where p_0 is obtained from the normalizing condition $\sum_{n=0}^{k}p_n = 1$ and is given as

$$p_0 = \left[\frac{1}{1+\sum_{n=1}^{k}\left[\frac{\lambda^n}{\prod_{r=1}^{n}\left(\mu+\frac{\nu(\nu^r-1)}{\nu-1}\right)\prod_{r=0}^{n-1}(a-rb)}\right]}\right]$$

An important measure of system performance is the average number of customers in the system, which is denoted by *L*. Let *P(s)* be the p.g.f of the steady state probability. Then we note that

$$L = P'(1) = \frac{d}{ds}P(s)\,|_{s=1}$$

By solving this, we get

$$P'(s) = \frac{\lambda}{\mu}\left[\sum_{n=0}^{c-1}p_nS^n+\sum_{n=c}^{k-1}\frac{p_nS^n}{(a-(n-c)b)}+\sum_{n=c+1}^{k}(n-c)p_nS^{n-1}\right] \quad \text{…(5)}$$

Putting S=1 in equation (5), we get the average number of customers in the system i.e. *L is*

$$L = P'(1) = \frac{1}{\left(\mu-\frac{\nu}{\nu-1}\right)}\left[\sum_{n=0}^{k-1}\frac{(n+1)p_n}{(a-nb)}-\frac{\nu}{\nu-1}\sum_{n=1}^{k}np_n\nu^{n-1}\right] \text{…(6)}$$

Also the average number of customers in the queue i.e. L_q is

$$L_q = \sum_{n=2}^{k}(n-1)p_n = \sum_{n=2}^{k}\left[\frac{(n-1)\lambda^n}{\prod_{r=1}^{n}\left(\mu+\frac{\nu(\nu^r-1)}{\nu-1}\right)\prod_{r=0}^{n-1}(a-rb)}\right]p_0$$

The other performance measures like W, $W_q$are can be calculated by using the Little's formula.

5. NUMERICAL ANALYSIS

To illustrate the use of our results, we apply them to a queuing problem. For some standard values of "a" and "b" such that $k<\frac{a}{b}$, we can to see the fluctuations in the performance measures of our proposed model. Here we have considered the arrival rate λ is 36 and the service time μ is 12. Now for different balking probabilities with constant reneging rate and for k=5, we have the following results:

Table 1: Variations in performance measures for b=2

λ= 36, υ =3, μ=12, k=5		a = 20	a = 30	a = 40
b=2	Po	0.892831	0.925917	0.943393
	L	0.175321	0.11579	0.08179
	L_q	0.00003	0.000009	0.000004

We can see from Table 1 that the expected system size and queue size decreases while the probability of empty system increases with the increases of balking probabilities (i.e. increasing 'a' keeping 'b' constant) which must happen. That is, the results are consistent as per model.

Table 2: Variations in performance measures for b=3

λ= 36, υ =3, μ=12, k=5		**a = 20**	**a = 30**	**a = 40**
b=3	Po	0.892829	0.925917	0.943393
	L	0.177616	0.112231	0.082053
	L_q	0.000033	0.000009	0.000004

Similarly, we have seen from Table 2that the average queue and system length decreases while the probability of no customers in the system increases with the decreases of joining probability i.e. by increases the balking probabilities (i.e. increasing 'a' keeping 'b' constant) which also must happen. That is, the results are also consistent as per our proposed model.

Table 3: Variations in performance measures for b=4

λ= 36, υ =3, μ=12, k=5		**a = 20**	**a = 30**	**a = 40**
b=4	Po	0.892827	0.925917	0.943393
	L	0.180198	0.112922	0.082332
	L_q	0.00003	0.000009	0.000004

In this case also we have seen from Table 3that as increasing 'a' keeping 'b' constant i.e. decreasing the joining probability, the average queue and system length decreases. Also the probability of no customers in the system increases with by increases the balking probabilities which are expected.

Therefore by combining the all results given above tables, we can conclude that as increasing the values of the parameters "a" and "b", the average system length is decreases i.e. as the balking probabilities increases keeping the reneging rate as constant, the expected system will decrease as well as average queue length will also increase which is highly expected. In these situations, the probability of joining of an arriving customer in that system will also decrease. Similarly by increasing the parameter "b" keeping "a", we have seen that expected system length and queue length increases simultaneously where the probabilities of no customer in the system decreases in this case which obvious from our practical point of view.

6. CONCLUSION AND FUTURE WORK:

The stationary probabilities of system size of a finite capacity Markovian queuing system with one server, reverse balking, and reneging are obtained. The performance measures like expected system size, average queue length etc. are also derived.

The paper may be extended to one or more heterogeneous servers. Other performances measures can also be perform in future work. The economic aspects of the model can also be explored.

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